PHASE TRANSITIONS OF THE LOGISTIC MAP WITH INHOMOGENEOUS NOISE

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Noisy alias random dynamical systems in recent years have attracted the interest of physicists and mathematicians, for two basic reasons: Physically, they add a good deal of realism to the theoretical models of many natural phenomena which were hitherto modeled by deterministic dynamical systems, since it has been realized that noise is almost ubiquitous in reality and often significantly affects the behaviour of complicated systems. Mathematically, random dynamical systems present an attractive synthesis of the fields of stochastics and general dynamical systems in which many branches of mathematical physics and pure mathematics converge and generate new insights. In both disciplines, it soon became clear that random dynamical systems possess a number of interesting new properties, most prominently the phenomena *noise induced stability*, *on-off intermittence*, and *stochastic bifurcations* also called *noise induced (phase) transitions*.

We consider the special noisy system

$$x_{t+1} = f_{\rho,\xi_t}(x_t) = \rho x_t(1-x_t) + \xi_t(x_t - x^*) \mod 1,$$

where ξ_t is an i.i.d. random variable in the interval $[-\sigma/2, \sigma/2]$, and $x^* = (\rho - 1)/\rho$ is the unique nonzero fixed point of the logistic map $f_\rho = f_{\rho,0}$. We show that the special inhomogeneous configuration of the noise pinched to the fixed point makes the system amenable to a basic theoretical analysis. Especially, we find that there exists a range of parameter values (ρ, σ) which we call the **stable phase**, and for which the system's long-time behaviour becomes stabilized at the fixed point x^*. We experimentally establish the existence of this phase by finding a perfect match of the theoretical prediction of the *Lyapunov exponent* with it's value approximated by numerical simulation. We show that various types of noise induced transitions occur at the border of the stable phase. One of these proves to be a stochastic bifurcation and we analyze it by further experimentation, showing that it bears a strong resemblance to the phenomenon of *symmetry breaking phase transitions* in statistical physics. We propose a physically relevant *order parameter* for this transition via the *return time statistics* and try to determine the *critical exponent* of this parameter.

Finally, we sketch how the special noisy dynamical system considered arises in the game-theoretical context of learning and decision making under uncertainty. Here, it is a very simple example of a model for prejudiced behaviour.

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